SEM-II
Hons (C IV: WAVES AND OPTICS)
Fraunhofer diffraction: Single slit, Double slit and Multiple slits

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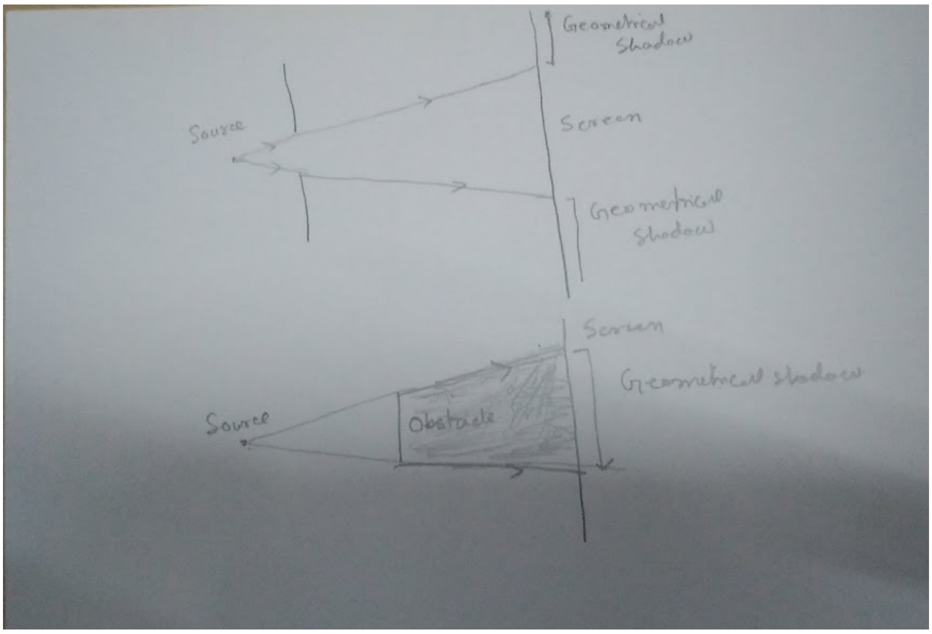
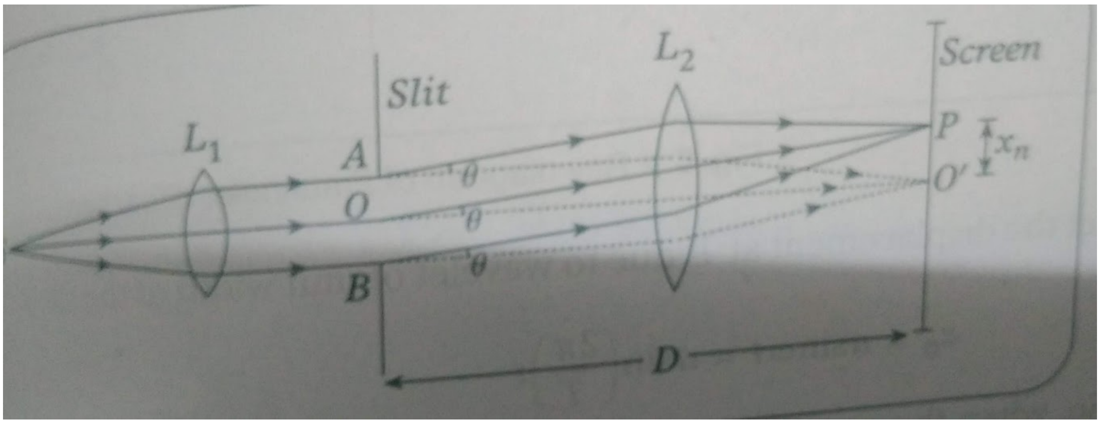
In this lecture I will start diffraction of light; here first I will explain what is diffraction? And then I will describe single slit, double slit and multi slit diffraction pattern.
The bending of light around the sharp corners of an obstacle or slit and spreading into the regions of the geometrical shadow is called diffraction of light. In principle all types of waves i.e. radio waves ,sound waves show diffraction effect.

The diffraction of light is divided into two class
a. Fresnel Diffraction b. Fraunhofer Diffraction.

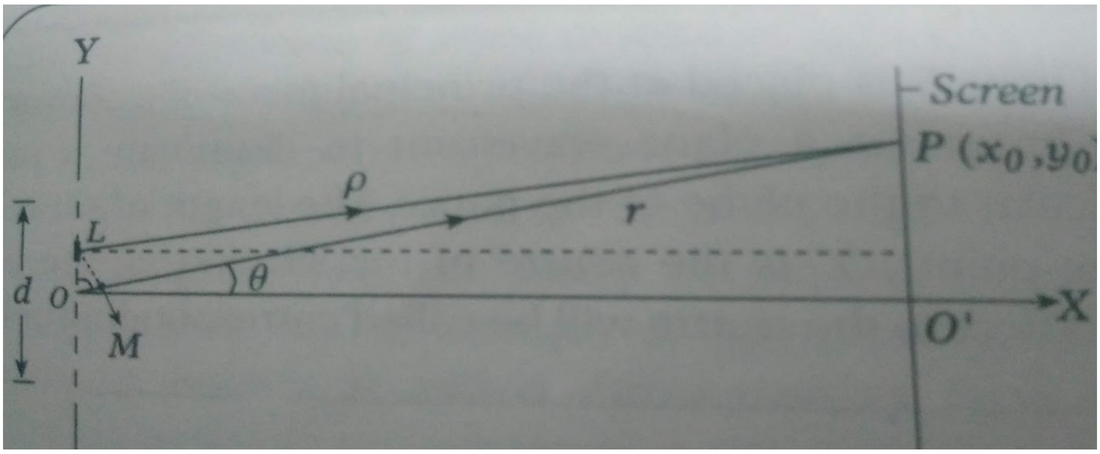


1 Fraunhofer Diffraction:

In this lecture season we are interested only Fraunhofer Diffraction pattern:
Let us consider a monochromatic source of light S is placed at principle focus of a convex lens $L1$ and AB a narrow rectangular slit. The point o is the center of the slit and all points symmetrically situated with respect to the center will be called corresponding points.



Accroding to the Huygen's principle when the plane wavefront occupies the plane of the slit , all the particles in the aperture become secondary source of disturbance. Each of such vibrating particle sends out waves. Some disturbances will move in the original direction and some are deviated at angle θ. The undeviated secondary disturbance will be brought to the focus at the point$o$ on the screen by another convex lens$L2$. In the figure central maximum at point o' for undeviated disturbances. The disturbances those diffracted through an angle θ will form another parallel beam and this will be brought to the focus to an another point$P$ on the screen.



We can consider the slit width d is divided into a large number of infinitesimal elements of equal widths dy
Let the displacement at P due to the wavelet of unit width of the slit at o is

$$z = aSin\omega t \tag{1.1}$$

$$= aSin(\frac{2\pi}{T})t \tag{1.2}$$

Hence the displacement at P due to the element dy at any time t is given by

$$dz = adySin2\pi(\frac{t}{T} - \frac{\rho}{\lambda}) \tag{1.3}$$

If d is the width of the slit , the total disturbance at P due to the whole wavefornt

$$z = \int_{-\frac{d}{2}}^{\frac{d}{2}} dz \tag{1.4}$$

$$= a\int_{-\frac{d}{2}}^{\frac{d}{2}} Sin2\pi(\frac{t}{T} - \frac{\rho}{\lambda})dy \tag{1.5}$$

$$= a\int_{-\frac{d}{2}}^{\frac{d}{2}} Sin2\pi(\frac{t}{T} - \frac{r - ySin\theta}{\lambda})dy \tag{1.6}$$

$$= a\int_{-\frac{d}{2}}^{\frac{d}{2}} Sin2\pi(\frac{t}{T} - \frac{r}{\lambda} - \frac{ySin\theta}{\lambda})dy \tag{1.7}$$

$$= (ad\frac{Sin\alpha}{\alpha})Sin2\pi(\frac{t}{T} - \frac{r}{\lambda}) \tag{1.8}$$

Where $\alpha = \frac{\pi dSin\theta}{\lambda}$

$$= A_\theta Sin2\pi(\frac{t}{T} - \frac{r}{\lambda}) \tag{1.9}$$

Where $A_\theta = ad\frac{Sin\alpha}{\alpha}$
So the resultant disturbance is also a simple harmonic vibration with amplitude A_θ.
Now the intensity at P is

$$I = A_\theta^2 = a^2d^2\frac{Sin^2\alpha}{\alpha^2} \tag{1.10}$$

$$= I_0\frac{Sin^2\alpha}{\alpha^2} \tag{1.11}$$

Where $I_0 = a^2d^2$ is the maximum intensity when $\theta = 0$ i.e. $\alpha = 0$.

The position of maxima and minima

I. Central maximum : When $\theta = 0$ i.e. $\alpha = 0$
$I_0 = a^2d^2$, So the central maximaum will be obtained at $\theta = 0$ i.e. at the point o' on the screen.

II.Secondary minima: These are seen for $Sin\alpha = 0$
i.e. $\alpha = n\pi$ Where $n = 1, 2, 3, ...$
or $dSin\theta = n\lambda$ is the condition for secondary minima(since $\alpha = \frac{\pi dSin\theta}{\lambda}$).

III. Secondary maxima: These are seen for $Sin\alpha = \pm 1$ So

$$\alpha = (2n \pm 1)\frac{\pi}{2} \tag{1.12}$$

$$or \ \ dSin\theta = (2n \pm 1)\frac{\lambda}{2} \tag{1.13}$$

is the condition for secondary maxima. The different values of α corresponding to $1st$order maxima and $2nd$ order maxima is equal to $\pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, ...$ respectively.

If the intensity I_1 of the $1st$ order secondary maxima is

$$I_1 = I_0\frac{Sin^2\frac{3\pi}{2}}{\frac{3\pi}{2}^2} \tag{1.14}$$

$$= \frac{I_0}{22} \tag{1.15}$$

Similarly If I_2 intensity of the $2nd$ order secondary maxima i.e.$\alpha = \frac{5\pi}{2}$

$$I_2 = I_0 \frac{Sin^2 \frac{5\pi}{2}}{\frac{5\pi}{2}^2} \tag{1.16}$$

$$= \frac{I_0}{62} \tag{1.17}$$

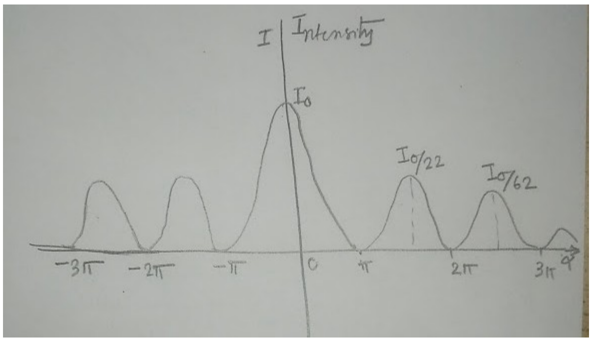


Figure 1: Intensity distribution for single slit diffraction pattern.

So it is found that
1.The diffraction pattern consists of a central bright maximum at o' followed by alternate secondary maxima and minima on the either side of it.

2.The width of the central maximum is twice as that of of a secondary maximum.

3.The intensity of the secondary maximum goes on decreasing with the increasing of the order of the maximum.

Position of secondary maxima:

$$I = I_0 \frac{Sin^2\alpha}{\alpha^2} \tag{1.18}$$

In order to determine position of secondary maxima , we differentiate the above equation w.r.t α and set equal to zero.

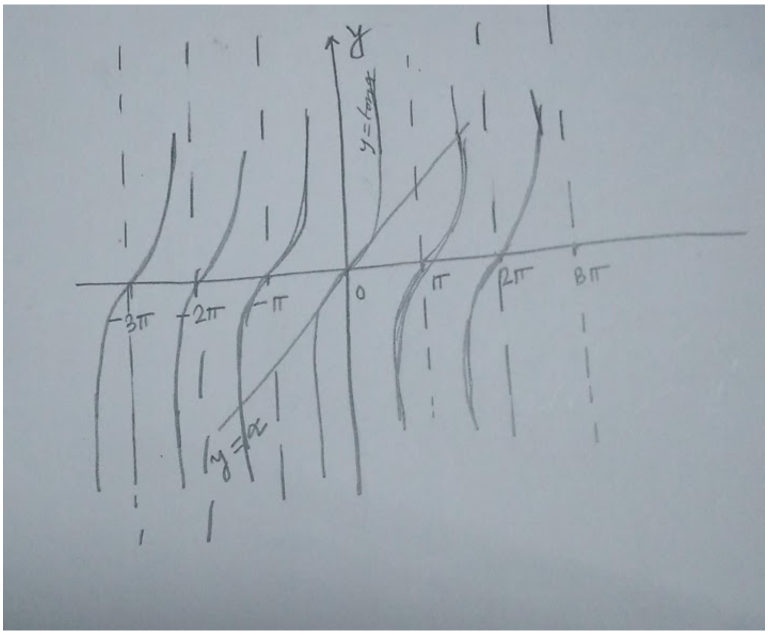
$$\frac{dI}{d\alpha} = I_0[\frac{2Sin\alpha Cos\alpha}{\alpha^2} - \frac{2Sin^2\alpha}{\alpha^3}] = 0 \tag{1.19}$$

or

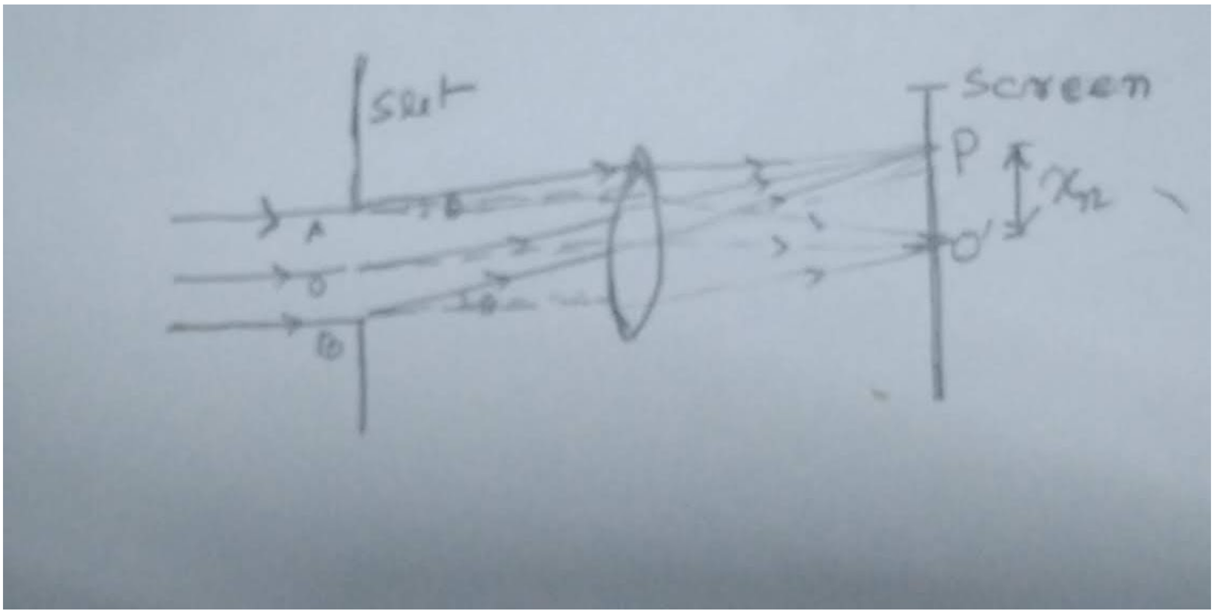
$$\alpha - tan\alpha = 0 \tag{1.20}$$

$$tan\alpha = \alpha \tag{1.21}$$

This equation gives the position of maxima. The root $\alpha = 0$ corresponds to central maximum. If the graph are plotted $y = tan\alpha$, it seen that the point of intersection $(\alpha = \pm 1.43\pi, \alpha = \pm 2.46\pi)$ etc.



Width of a secondary maximum and minimum:



If x_n is the distance of the nth minimum from the center of the screen and D is the distance between the slit and the screen then

$$Sin\theta_n = \frac{x_n}{D} \tag{1.22}$$

$$dSin\theta_n = n\lambda \tag{1.23}$$

So

$$\frac{x_n}{D} = \frac{nD\lambda}{d} \tag{1.24}$$

or

$$x_n = \frac{n\lambda}{D} \tag{1.25}$$

In this situation $D = f$(Focal length of lens $L2$)

So the fringe width of two consecutive minima is

$$\beta = x_{n+1} - x_n \tag{1.26}$$
$$= (n+1)\frac{D\lambda}{d} - \frac{nD\lambda}{d} \tag{1.27}$$
$$= \frac{D\lambda}{d} \tag{1.28}$$

For maximum

If x'_n is the distance of nth maximum from the central maximum o',then

$$Sin\theta_n = (2n+1)\frac{\lambda}{2d} \tag{1.29}$$
$$\frac{x'_n}{D} = (2n+1)\frac{\lambda}{2d} \tag{1.30}$$
$$x'_n = (2n+1)D\frac{\lambda}{2d} \tag{1.31}$$

So the width of the secondary maximum

$$\beta' = x'_{n+1} - x'_n \tag{1.32}$$
$$= \frac{D\lambda}{2d}(2n+2+1) - \frac{D\lambda}{2d}(2n+1) \tag{1.33}$$
$$= \frac{2D\lambda}{2d} \tag{1.34}$$
$$= \frac{D\lambda}{d} \tag{1.35}$$

Hence the width of the secondary maximum is same as that of a secondary maximum i.e. $\beta' = \beta$.

Width of central maximum:

The central maximum extends upto a distance of first order minima on both sides of the center of the screen i. e. from central maximum.
so the width of the central maximum ,

$$\beta_0 = 2x_1 \tag{1.36}$$
$$= 2\frac{D\lambda}{d} \tag{1.37}$$
$$\beta_0 = 2\beta. \tag{1.38}$$

Problem-1. A screen is placed at a distance of 90 cm from a narrow slit. The slit is illuminated by a parallel beam of light of wavelength $6000A^0$. Calculate the width of the slit if the first minimum is at a distance of $1mm$ on either side of the central maximum.

Problem-2. Find the angular width of the central bright fringe in the Fraunhofer diffraction pattern of a slit of width $0.24mm$. Wavelength of the light used is $5000A^0$.